



Using the Relation Between Quantity, Cost and Price to Increase Company Profit under Existing Production Capacity

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ABSTRACT

Small and medium enterprises are constantly facing problems when looking to increase sales and profits to benefit from economies of scale, managing the complex relation between quantity, price, unit total cost and unit fixed costs. The price, costs and quantities are interconnected and they influence each other, considering the competition, the response of the clients and the need for a better use of existing production capacities. This paper offers the managers a solution to the complex issue of correlating the fore-mentioned variables in their efforts of increasing total operating profit. We have identified the correlations managers should observe in their attempts to maximize profits under the constraints of existing production capacity.

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1. Introduction

Small and medium industrial companies should be looking to increase their sales in quantitative terms to benefit from the economies of scale, managing the complex relation between price, unit total cost, unit fixed cost. The price, costs and quantities are interconnected, such as any change in the level of one variable affects the levels of the other measures, given the competition in the market, the response of the demand (clients) and of company competitors as well as the need of a better use of the existing production capacities. As such, the managers of small and medium industrial companies are faced with complex issues when they are trying to increase their profits, correlating the discount policies (designed to support their clients buy more units), the evolution of unit total and fixed costs and of quantities sold.

Many financial problems of small and medium industrial companies (SME's) originate in the incomplete use of their existing production capacities. The inability of using to the fullest their existing production capacity partially impairs SME's competitiveness in their market segments due to higher prices, lower margins and reduced room for maneuver in their relations with suppliers, clients and competitors.

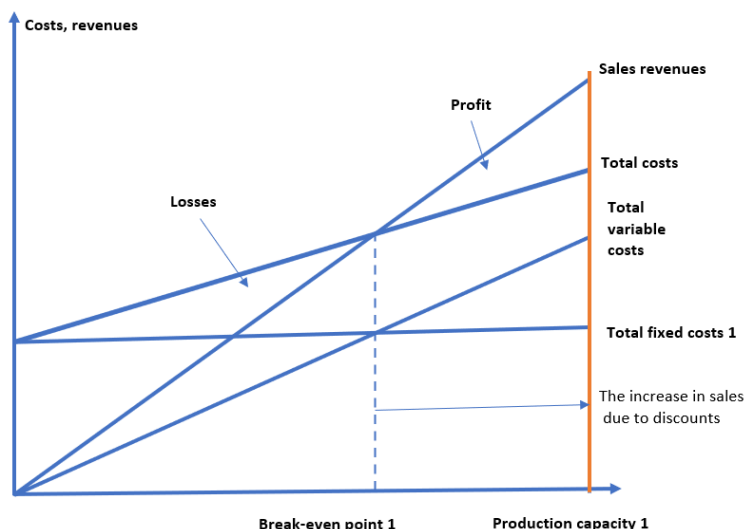


Figure 1.

Source: the author

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An essential way of increasing sales, efficiency and profits is reaching the maximum potential of existing production capacities, which allows SME's to benefit from economies of scale, minimize their costs and optimize the relation between sales, price policies and total profits. The situation is described in graph no. 1.

Even if for corporations and big companies the objective of profit maximization/shareholders' wealth maximization has lost its appeal and prevalence, being replaced with objectives such as stakeholder wealth maximization, sustainable increase in company value, for small and medium companies the sustainable profit maximization remains a main purpose and focus, obviously considering the current trends and regulations of environmental protection, eco and community friendly approaches (Pizer, William, A., 2002; Friede, G., Busch, T., & Bassen, A. 2015).

2. Objectives

This paper identifies the relations between significant variables, such as quantity, unit variable costs, unit fixed costs and price, which will allow managers set their decisions based on objective references and to calibrate/recalibrate the decisions according to new data available from suppliers, production and sales. The main task of the managers is to make sure that company unit costs are divided correctly into variable unit costs and fixed costs.

The relations established will allow the SME's managers find either the proper discount (the reduction in initial price) or the necessary increase in the quantitative volume of sales, thereby ensuring a favourable result of their actions of increasing the sales and making use of scale economies.

The main measure used to express the success of the managerial actions of increasing the quantitative volume of sales is the marginal operating profit, respectively the positive difference between the operating profit obtained as a result of company's actions and the level of operating profit which was previously registered. The increase of sales turnover alone is not sufficient, since does not guarantee an increase in operating profit in case of a decrease of the profit margin.

We express operating profit as a product between quantities and unit profit margin, as an attempt to reduce multicollinearity between dependent variables.

3. Material and methods

We will use the indexes (Vasilescu, Gh., Niculescu, I., Wagner, Fl., Zaharia, O. 1997; Dincă, M.S., Dincă G., 2011, Dincă, M.S., 2007) method to express the relation between total operating profit, respectively unit profit margin and the independent variables of quantities sold, overall unit cost, variable unit costs, fixed unit costs and price.

In most cases, the SME's offer discounts (reduction in selling prices) when clients buy larger quantities of their products and as such the new selling price (p_1), resulted from this action is lower than the previous selling price (p_0), or $p_1 < p_0$.

The new selling price can be higher than previous price (when the demand for the respective product is growing and it has an inelastic character), equal to that (when the demand for the respective product is growing and it has an elasticity equal to one) or lower than previous price (which is the most common situation).

The companies offer discounts to increase their quantitative sales from the previous level of q_0 to the new level, q_1 and hence the index of quantities sold is higher than one ($I_q > 1$; Kelly, David, L. 2005; Krysiak, Frank, C. 2008).

The effect upon the new level of company sales (Ca_1) is contingent upon the relation between the evolution of the indexes of quantities, respectively price, since the index of sales is the product between the index of quantities and the index of price (Dincă, M.S., Dincă G., 2011):

$$I_{Ca} = I_q \cdot I_p$$

To obtain an increase in sales ($Ca_1 > Ca_0$) the index of sales turnover has to be higher than one ($I_{Ca} > 1$), which in turn provides the solution for how company unit sales (quantities sold) should increase when they offer a certain discount (which generates a lower than one index of prices, $I_p < 1$, since the company lowers its selling price, or $p_1 < p_0$).

This implies quantities should increase a higher pace, to secure an increase in sales turnover when the company offers a discount to support its clients buy more and reach its maximum production and sales capacity, making use of the scale economies:

$$I_q > \frac{1}{I_p}$$

For example, if the company offers a 10% discount to support a higher volume of units bought by its clients, it should make sure that its quantities sold will increase by at least 11.11% to compensate the lower selling price. If the discount is of 5%, the minimal increase in quantities sold should be 1/0.95, or 5.26%.

However, the increase in sales turnover does not guarantee the company will obtain an increase in its profits as a result of the new price policy (offering of discounts to support economies of scale and reach 100% of the existing production and sales capacity; Vujevic, I., 2005).

It is possible that even if sales turnover increase, the company registers losses or at least a diminished self-financing capacity, which is needed to co-finance its upcoming investments in non-current production assets to accommodate further increases in demand.

This is why we need to study the impact upon company total operating profit (Dincă, M.S., Dincă G., 2007; Chen, C.H. and Lu, C.L., 2011). The total operating profit can be expressed as a product between quantities sold and the profit margin (Weitzman, M.L. 1974) and its evolution determined using its index, which should be higher than one to register an increase in the current period (after offering the discount) as compared to previous period (before offering the discount):

$$Pr = q \cdot m = q(p - c) = q(p - cv - cf)$$

$$I_{Pr} > 1, \text{ or } I_q \cdot I_m > 1$$

Since usually quantities sold increase as a result of discount offering it implies that the level of profit margin should increase, remain the same as in the previous period or even decrease, provided that the respective decrease complies with the condition $1 > I_m > \frac{1}{I_q}$ (Paxson, D. and Pinto, H., 2005).

For example, if the quantities sold increase by 10%, the profit margin should not decrease by more than 9.1% (an index higher of 0.9090 yet lower than one). Of course, this is the minimal level, not the desired one, since the company has to control for the self-financing capacity as well (not only for the existence of marginal operating profit). Increased self-financing capacity is required for the co-financing of new upcoming investments, needed to accommodate the further increases in demand for the company products.

The profit margins from before and after the granting of the discount (m_0 and m_1) are determined as:

$$m_0 = p_0 - cup_0$$

$$cup_0 = cv_0 + cf_0$$

$$cf_0 = \frac{CF_0}{q_0}; CF_1 = CF_0; cv_1 = cv_0$$

$$cf_1 = \frac{CF_0}{q_1} = \frac{cf_0}{\frac{q_1}{q_0}} = \frac{cf_0}{I_q};$$

$$m_1 = p_1 - cup_1 = p_1 - cv_1 - cf_1 = p_0 \cdot I_p - (cv_0 + \frac{cf_0}{I_q})$$

Where:

cup – overall unit cost;

cv_0 – variable unit cost from the reference period; cv_1 – variable unit cost after granting the discount;

cf_0 – fixed unit cost from the reference period; cf_1 – fixed unit cost after granting the discount;

CF – total fixed costs;

p – selling price, net of VAT;

I_p – the price index, respectively the ratio between p_1 and p_0 ;

I_q – the index of quantities sold.

3.1. Determining the profit resulting from granting the discount with a focus on unit profit margin

In this configuration we analyze only the evolution of unit profit margin, assuming that quantities sold always register an increase as a result of offering clients a discount. There are three situations, respectively what happens when the company increases the quantities sold and preserves the existing profit margin (first situation), increases quantities and the profit margin (the second situation), respectively increases quantities and decreases the margin, yet the increase in quantities more than compensates the decrease in profit margin (the third situation).

A. Increasing total operating profit by increasing quantities sold ($I_q > 1$), whilst maintaining the profit margin (Index of unit profit margin = 1)

The profit margin is expressed before (m_0) and after the granting of discount (m_1).

The previous profit margin (m_0) is given by:

$$m_0 = p_0 - cv_0 - cf_0$$

Hence, the current profit margin (m_1) is given by

$$m_1 = p_1 - cv_1 - cf_1 = p_0 \cdot I_p - cv_0 - \frac{cf_0}{I_q}$$

If we preserve the same level of the profit margin, it means the index of the profit margin will be equal to one, or $m_1 = m_0$ hence:

$$p_0 \cdot I_p - \left(cv_0 + \frac{cf_0}{I_q} \right) = p_0 - cv_0 - cf_0 \quad (1)$$

Rearranging the terms from equation no. 1 we get:

$$\begin{aligned} p_0(I_p - 1) &= cf_0 \left(\frac{1}{I_q} - 1 \right) \\ p_0(1 - I_p) &= cf_0 \left(1 - \frac{1}{I_q} \right) \\ \frac{cf_0}{p_0} &= \frac{1 - I_p}{1 - \frac{1}{I_q}} = \frac{disc}{\frac{I_q - 1}{I_q}} = \frac{disc}{\frac{\Delta\%q}{I_q}} \end{aligned} \quad (2)$$

Where:

$disc$ or $(1 - I_p)$ – the discount offered by the company;

$\Delta\%q$ – percentage increase of quantities sold;

I_q – the index of quantities sold.

The relation (2) can be solved for I_p , giving the price discount the company can grant to support the increase in quantities sold, whilst maintaining the same profit margin. The same can be done for I_q , giving the required increase in quantities sold needed to insure the preservation of profit margin.

Actually, solving for $(1 - I_p)$ or the discount the company should offer, we get:

$$disc = \frac{cf_0}{p_0} \left(1 - \frac{1}{I_q} \right) = w_0 \frac{\Delta\%q}{I_q} \quad (3)$$

Where:

w_0 – the weight of unit fixed costs into price from the previous period;

$\Delta\%q$ – the percentage increase of quantities

Solving for I_q , we get the increase in quantities sold needed to ensure a marginal operating profit by keeping the profit margin at the level from the base period. In this hypothesis the company wants to find what happens when it offers a certain price discount to its clients in an attempt to reach the maximum production capacity.

$$\begin{aligned} 1 - \frac{1}{I_q} &= \frac{1 - I_p}{\frac{cf_0}{p_0}} \\ I_q &= \frac{\frac{cf_0}{p_0}}{\frac{cf_0}{p_0} - (1 - I_p)} = \frac{w_0}{w_0 - disc} \end{aligned} \quad (4)$$

Where:

w_0 – the weight of unit fixed costs into price from the previous period;

$disc$ – the discount the company grants to its clients (or $1 - I_p$).

B. Increasing total operating profit by increasing both quantities sold ($I_q > 1$) and profit margin (Index of profit margin > 1)

As mentioned before, the index of total operating profit is equal to the product between the index of quantities and the index of the profit margin:

$$Index\ of\ total\ profit = Index\ of\ quantities \times Index\ of\ profit\ margin.$$

The increase of total profit is secured as in this assumption both quantities and profit margin will increase (their indexes are both higher than one).

The index of the profit margin (Im) can be expressed as:

$$Im = \frac{m_1}{m_0} = \frac{p_1 - cv_1 - cf_1}{p_0 - cv_0 - cf_0} = \frac{p_0 \cdot I_p - cv_0 - \frac{cf_0}{I_q}}{p_0 - cv_0 - cf_0} > 1$$

The increase of profit margin requires that the level from the current period should be higher than the level from before granting the discount, or an index higher than one, respectively:

$$\begin{aligned} p_0 I_q \cdot I_p - cv_0 \cdot I_q - cf_0 &> I_q (p_0 - cv_0 - cf_0), \\ p_0 (I_q \cdot I_p - I_q) &> cf_0 (1 - I_q) \\ p_0 I_q (I_p - 1) &> cf_0 (1 - I_q) \\ p_0 (I_p - 1) &> cf_0 \frac{1 - I_q}{I_q} \end{aligned}$$

Changing the sign as in this case $I_p < 1$ and $I_q > 1$, we will get relation no. 5:

$$\frac{I_q(1 - I_p)}{I_q - 1} > \frac{cf_0}{p_0} \text{ or } \frac{I_q}{\Delta\%q} \cdot disc > w_o \quad (5)$$

Where:

$\Delta\%q$ – percentage increase of quantities sold;

disc – the price discount offered by the company;

w_o – the initial weight of unit fixed costs into price.

Increasing the profit margin requires that product between the ratio of index and percentage increase of quantities and the price discount should be higher than the weight of unit fixed cost into price from the previous period.

Of course, the relation no. 3 can be solved for quantities, showing by how much the quantities sold should increase to secure an increase of profit margin (relation no. 6) or for the price/discount, in which case it specifies the maximum discount the company could offer (relation no. 7) to secure an increase of the profit margin under an estimated increase in quantities sold as a result of offering a discount to company clients.

$$I_q > \frac{\frac{cf_0}{p_0}}{\frac{cf_0}{p_0} - (1 - I_p)} > \frac{w_o}{w_o - disc} \quad (6)$$

$$disc < w_o \cdot \frac{\Delta\%q}{I_q} \quad (7)$$

C. Increasing total operating profit by having a bigger increase in quantities sold compared to the decrease of profit margin

In this situation the total operating profit will increase as a result of compensating the decrease of profit margin by a relatively higher increase of quantities sold. For example, the quantity sold increases by 10%, the profit margin decreases by 5% and total operating profit will increase by 4.5%, respectively 1.1×0.95 minus 1.

The decrease of profit margin would imply that the discount should be higher than the product between the weight of fixed costs into price from previous period and the ratio between percentage increase and index of quantities sold (relation 8 below):

$$disc > w_o \cdot \frac{\Delta\%q}{I_q} \quad (8)$$

3.2. Determining the profit resulting from granting the discount with an overall approach

The overall approach implies including the evolution of quantities and dealing with both factors (quantities sold and unit profit margin) in the same time.

As such the index of total operating profit will be:

$$I_{Pr} = I_q \cdot I_m = I_q \frac{p_1 - cv_1 - cf_1}{p_0 - cv_0 - cf_0} = I_q \frac{p_0 \cdot I_p - cv_0 - \frac{cf_0}{I_q}}{p_0 - cv_0 - cf_0} = I_q \frac{I_p \cdot I_q \cdot p_0 - I_q \cdot cv_0 - cf_0}{I_q(p_0 - cv_0 - cf_0)} \\ = \frac{I_p \cdot I_q \cdot p_0 - I_q \cdot cv_0 - cf_0}{p_0 - cv_0 - cf_0}$$

To secure an increase of total operating profit its index should be higher than one, respectively the numerator should be higher than the denominator of the ratio:

$$I_q \cdot I_p \cdot p_0 - I_q \cdot cv_0 - cf_0 > p_0 - cv_0 - cf_0$$

After further processing of the relation, we arrive to:

$$(I_q \cdot I_p - 1) \cdot p_0 > (I_q - 1)cv_0$$

or

$$\frac{I_q \cdot I_p - 1}{I_q - 1} > \frac{cv_0}{p_0}, \text{ or } \frac{I_{Ca} - 1}{I_q - 1} > wv_0 \quad (9)$$

Where:

I_{Ca} – index of sales turnover, or the product between the index of quantities sold and the index of price;

wv_0 – the weight of variable unit costs into price from the reference period.

Relation no. 9 shows that total operating profit will increase if the ratio between the evolution of sales turnover and evolution of quantities sold will be higher than the weight of unit variable costs into price from the reference period.

However, since sales turnover is also influenced by quantities sold, we will solve inequation (9) for the two independent variables, respectively quantity and price, to find how each of the two should evolve to secure an increase of total operating profit.

Solving for quantities we have:

$$I_q(I_p \cdot p_0 - cv_0) - cf_0 > p_0 - cv_0 - cf_0 \\ I_q(I_p \cdot p_0 - cv_0) > p_0 - cv_0 \\ I_q > \frac{p_0 - cv_0}{I_p \cdot p_0 - cv_0} \text{ or } I_q > \frac{p_0 - cv_0}{p_1 - cv_0} \text{ or } I_q > \frac{mvc_0}{mvc_1} \quad (10)$$

Where:

mvc_0 – the margin of unit variable costs from the reference period;

mvc_1 – the margin of unit variable costs from the period after introducing the discount.

Relation no. 10 specifies that securing an increase of total operating profit implies that the quantities sold should increase at a higher pace that the ratio between previous and current unit variable cost margin. Obviously in this case we know the discount offered by the company and we find the necessary increase in quantities sold.

Solving for price we get:

$$I_q(I_p \cdot p_0 - cv_0) > p_0 - cv_0 \\ I_p I_q p_0 > p_0 - cv_0(1 - I_q) \\ 1 > I_p > \frac{p_0 - cv_0(1 - I_q)}{p_0 \cdot I_q} \quad (11)$$

Obviously, in case of the evolution of price, things look a little more complicated than for quantity, since the index of price should be higher than the right term of the inequation yet lower than 1, since usually the companies grant discounts to increase their quantities sold.

4. Discussions and conclusions

The problem of ensuring an increase of total operating profit as a result of company efforts in view of reaching the maximum potential of existing production and sales capacities and making use of scale economies can be addressed in two configurations.

In the first approach we consider that as a result of the discount policy, quantities sold will increase with certitude, which moves the analysis only to the evolution of unit profit margin. Under a certain increase of quantities sold, the unit profit margin can be kept unchanged, increase or even decrease (provided that the decrease of profit margin is compensated by a higher increase in quantities sold).

In the second configuration the analysis is done for overall operating profit (considering the evolution of both quantities and profit margin), without assuming implicitly that quantities sold will increase.

The overall approach obviously gives slightly different results (yet not contradictory ones), since the conditions set in this case are not confined only to profit margin but to the overall operating profit.

A significant difference between the two approaches is that in the first case the conditions refer to the weight of fixed costs into price, whereas in case of the overall operating profit approach the reference are variable unit costs (with their weight into price and their margin).

The overall approach seems more suited and it is congruent with the relations set in case of determining the marginal profit from granting commercial credit, in which case variable costs (via their weight into price and their margin) are also instrumental (Dincă, M.S., Dincă G., 2013).

Managers can employ these relations to calibrate/recalibrate their commercial credit policies and to set targets related either to the evolution of quantities sold under a given discount or to the discount they can offer under a certain estimated increase of quantities sold or even for the overall operating profit, without considering separately quantity and price evolutions.

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