



Exploring Advanced GARCH Models for Analyzing Asymmetric Volatility Dynamics for the Emerging Stock Market in Hungary: An Empirical Case Study

Shreevastava Aman^{*}, Raza Shahil^{**}, Bharat Kumar Meher^{***}, Ramona Birau^{****},
Anand Abhishek^{*****}, Mircea Laurentiu Simion^{*****}, Nadia Tudora Cirjan^{*****}

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ABSTRACT

The study was conducted on BUX Index volatility for the post-2008 (from 2011) global financial crisis period using advanced GARCH models (GARCH, TGARCH, EGARCH, IGARCH, PARCH, APARCH). Based on parameters and test results appropriate model was chosen (APARCH (1,1) at Student's t distribution) to study volatility, the presence of asymmetry, leverage effect, volatility clustering, and decay factor. Test results were linked with the macro and microenvironment (Political instability, Geopolitical crisis, Unemployment, Inflation, COVID-19, Slowdown, etc.) of the index and unique features of the index have been discovered (Like a sudden huge dip between 2020-2022). The study through mathematical and econometrics terms establishes causal links among the variables affecting the index. The paper is relevant to both investors and policymakers as BUX is one of the most important indicators as far as stock market in Hungary is concerned.

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1. Introduction

The Budapest Stock Exchange (BSE) stands as a pivotal arena for economic indicators, reflecting Hungary's financial landscape through its flagship index, the BUX. Comprising approximately 25 blue-chip Hungarian companies, the BUX serves not only as a barometer of the stock market's performance but also as a mirror reflecting the country's economic health. Traded on the Budapest Stock Exchange, the BUX is a weighted index, employing free-float capitalization weighting to gauge the average price change of shares with the highest market value and turnover in the equity sector. This methodology positions it as a pioneer among indices, rendering it an essential tool for investors seeking insights into Hungary's economic trends. Since its inception on January 2, 1991, the BUX has weathered diverse economic and geopolitical storms, with its trajectory influenced by both local and global events. Over the past decade and a half, Hungary has experienced significant geopolitical shifts, each leaving an indelible mark on the BUX's pricing dynamics. Notably, after the 2008 financial crisis, a series of global events, including the Arab Spring, the Paris Agreement, and the emergence of the COVID-19 pandemic, reshaped the economic landscape worldwide. Hungary, too, faced its share of challenges, with credit downgrades in 2011 owing to political instability and financial dependence on the national bank.

Subsequent years saw sluggish growth rates, punctuated by interventions from the European Union that catalyzed economic resurgence. However, the COVID-19 pandemic dealt a severe blow to Hungary's economy, precipitating a GDP slowdown, soaring national debt, and unprecedented levels of inflation. Against this backdrop, the BUX index exhibited heightened volatility, mirroring the uncertainties and disruptions plaguing global financial markets.

Investment decisions, inherently influenced by micro to macro factors encompassing corporations, industries, and broader economic trends, necessitate a nuanced understanding of market volatility. Herein lies the significance of ARCH-GARCH (Autoregressive Conditional Heteroscedasticity) analysis, a cornerstone in quantifying and forecasting volatility. Traditional methods such as historical volatility and Bollinger bands offer rudimentary insights accessible to novice investors. However, there are certain modern methods include many algorithmic functions for modeling volatility like ARCH, IGARCH, GARCH, EGARCH, TGARCH, PARCH,

^{*}, ^{**}, ^{***} PG Department of Commerce&Management, Purnea University, Purnea, India, ^{****} University of Craiova, "Eugeniu Carada" Doctoral School of Economic Sciences, Craiova, Romania, ^{*****} PG Department of Economics, Purnea University, Purnea, India, ^{*****} University of Craiova, "Eugeniu Carada" Doctoral School of Economic Sciences, Craiova, Romania, ^{*****} National Agency for Fiscal Administration (ANAF), Regional Directorate General of Public Finance Craiova. E-mail addresses: amanshreevastava1998@gmail.com (S. Aman), researchshahil@gmail.com (R. Shahil), bharatraja008@gmail.com (B. K. Meher), ramona.fbirau@gmail.com (R. Birau – Corresponding author), abhi2eco@gmail.com (A. Abhishek), simionmirceaurentiu@gmail.com (M. L. Simion), dascalunadia@yahoo.com (N. T. Cirjan).

APARCH, FIGARCH, MGARCH models, etc. These volatility models are used to describe a changing, possibly volatile variance and to predict the volatility of time series data (Meher, Birau, et al., 2023).

In light of these considerations, this research endeavors to explore advanced GARCH models to decipher the asymmetric volatility dynamics inherent in Hungary's BUX market index. By leveraging statistical time series analysis, this study aims to unravel the intricate interplay between past geopolitical events and the BUX's volatility, culminating in the development of a robust predictive model. Through this endeavor, we seek to furnish investors and policymakers with actionable insights into navigating the turbulent waters of Hungary's financial markets, thereby fostering informed decision-making and sustainable investment practices.

2. Literature review

An ample amount of work has been done around the volatility of Stock Indices with various econometrics and mathematical models and its measurement becomes important for the implementation of most economic or financial theories that guide such investment. Volatility is also important for evaluating the quality of performance of financial markets, with very volatile markets being assumed as not functioning effectively as a way of channeling savings into investment. Despite the significance of volatility, it is not an easy task to measure it, and there are many approaches to do that. (Park & Linton, 2012) A good volatility model must be able to capture not just observed facts but also several stylized facts (that are more than conditional variance). There is a need for a model to forecast 100 or even 1000 steps in the future due to long memory and fractionally integrated processes. (Engle & Patton, 2007) 330 ARCH-type models were studied in terms of their ability to predict the conditional variance. The models are compared out-of-sample through DM-\$ exchange rate data and IBM return data, where the IBM Data is based on a completely new data set of realized variance. No evidence was found that a GARCH(1,1) was outperformed by any of the more sophisticated models in our analysis of exchange rates, however, the GARCH(1,1) is inferior to models that can accommodate a leverage effect in our analysis of IBM returns (Hansen & Lunde, 2001).

For convenience let's bifurcate the review of the literature in a few segments. Segments will include an Understanding of Volatility Modeling, ARCH, and GARCH-based models, APARCH model, Application of APARCH models, and Some Previous models-based studies on the BUX Index followed by knowledge gap and research question in a sequential order.

Understanding of Volatility Modeling

Most empirical studies have utilized the Autoregressive Conditional Heteroskedasticity (ARCH) models developed by Engle (1982) and later generalized by Bollerslev (1986), along with extensions of the ARCH see, for example, Baillie, Bollerslev, and Mikkelsen (1996) (Murinde & Poshakhwale, 2001) Stock market volatility in the East European markets of Hungary and Poland is studied using daily indexes. (Poshakwale & Murinde, 2001) The evolution of volatility models has been motivated by empirical findings and economic interpretations. (Hansen & Lunde, 2005) A vector autoregression with a multivariate GARCH component and performing a variety of diagnostic tests, the main empirical result is the existence of limited interaction. (Scheicher, 2001a) The BEKK model of multivariate volatility shows signs of return volatility spillovers from Hungary to Poland, but there are no volatility spillover effects found in the opposite direction. (Kasch-Haroutounian & Price, 2001) The three most common asymmetric volatility models include the EGARCH model of Nelson (1991), GJR-GARCH OG Glosten, Jagannathan, and Runkle (1993), and the APARCH model of Ding, Grange, and Engle (1993) (Kisinbay, n.d.) The financial econometric volatility is silent upon links between asset return volatility and its determinants. Instead, the focus is on modeling volatility and not underlying macroeconomic factors (Bollerslev et al., 2010) Volatility is one of the important quantities for an investor and any economic and financial theory is dependent upon it. (Park & Linton, 2012) asymmetry in volatility is rare in emerging and frontier markets; asymmetry in correlations concerns the Hungarian stock market; and the relationship between volatility and correlations is positive and significant in the majority of countries.

Thus, diversification benefits shorten during periods of higher volatility. (Baumöhl & Lyócsa, 2014) The first issue may be the autocorrelation in error terms. To handle this unwanted situation the lagged dependent variables can be treated as independent variables in the mean equation. (Chand et al., 2012) Volatility surges in periods of economic or political instability or in the influence of certain factors. (CAMELIA CATALINA, 2019) Asymmetric volatility is discovered in crude oil prices due to the spread of COVID-19. The news related to the spreading of the pandemic COVID-19 has a significant impact on the price volatility of crude oil. (Meher et al., 2020) The presence of the heteroscedasticity test, a key element in understanding stock market volatility, was done using the ARCH Lagrange Multiplier (LM) test (Meher et al., 2024) A quantitative approach based on modeling stock volatility, and asset returns lead to the identification of investment opportunities about the international diversification of the portfolio. (Trivedi et al., 2021) Reliable forecasts can guide investors, helping them allocate their resources effectively and dodge potential losses. This, in turn, enhances financial literacy and stability among the larger masses. (Meher et al., 2024a) The channels through which the global financial turmoil affects developing countries may be sorted into financial channels and real channels. Financial

channels (direct effects) include the effects of stock markets, the banking sector, and foreign direct investment. Real channels (indirect effects) include effects through remittances, exports, imports, terms of trade, and aid. (Massa & Willem Te Velde, 2008) The Monte Carlo experiment reveals that if the model is wrongly specified, then commonly used unit root tests will misclassify inflation as a nonstationary, rather than a stationary process and the relationship between inflation and uncertainty that higher uncertainty leads to surged inflation persistence. (Canepa, 2024a) Enough has been briefly touched about volatility and a snapshot of models old to new has been provided let's move to the next section.

ARCH and GARCH-based Models

The GARCH model is an extension of the ARCH (Autoregressive Conditional Heteroskedastic) process, allowing for a more flexible lag structure to capture changing variance over time which is essential for modeling time series exhibiting long memory. (Bollerslev et al., 2010) The t -distributed statistics of standardized returns generally lead to a better average performance than the Gaussian. (Hansen & Lunde, 2005) To address the limitations of ARCH models, particularly the potential for negative variance estimates, Bollerslev (1986) and Taylor (1986) proposed a more parsimonious alternative: the Generalized ARCH (GARCH) model. GARCH generalizes the ARCH framework by incorporating lagged conditional variances, alongside past squared errors, into the equation for conditional variance. (Kumar, 2019) (Dickey & Fuller, 1979) provides valuable insights into the behavior of estimators in the presence of a unit root, contributing to the understanding of time series analysis and statistical inference. Exploring GARCH-MIDAS models for mixed-frequency data and expanding the control variables used (like higher-frequency indicators for financial liberalization) (Hartwell, 2014) Diagnostic tests confirmed the ARIMA-GARCH models were suitable for predicting returns and volatility of most indexes, except potentially the PX-50 which may require further examination. (Vošvrda & Žikeš, 2004) GARCH-EVT-Copula, for estimating liquidity-adjusted value-at-risk (L-VaR) of energy stocks. The results show that the GARCH-EVT-Clayton copula is the most effective in forecasting L-VaR compared to other models. This has implications for investors, market makers, and daily traders who consider liquidity in market risk computation. (Kamal & Paul, 2024) A new GO-GARCH-EVT-copula method estimates minimum risk hedge ratios better than existing models, likely due to its ability to capture data characteristics more effectively. (Karmakar & Sharma, 2020) The stock market is influenced by global sentiment and characterized by extreme values and linear and nonlinear variables. High-frequency data is important in accurately predicting stock market fluctuations. The extreme value theory (EVT) approach is reliable in detecting extreme values in univariate cases. A hybrid model combining EVT and machine learning (ML) can accurately estimate investment risk using multivariate and high-frequency data. (Melina et al., 2024) Standard GARCH models fail to account for this asymmetry. To address this limitation, the passage introduces more advanced models like GJR-GARCH (Glosten, Jagannathan, & Runkle, 1993), also known as Threshold GARCH (TGARCH), and EGARCH. These models explicitly incorporate the leverage effect, allowing for differential impacts of positive and negative shocks on the conditional volatility estimates. (Meher, Birau, et al., 2023) Not going into much depth of the ARCH-GARCH Universe we have taken a chronological tour of the GARCH family till the most recent development in the discipline. Let's delve into the Asymmetric Power ARCH model in the next section.

APARCH Model

The models that fit very well in the IBM return data are those that can accommodate a leverage effect, and the best workability is achieved by the A-PARCH(2,2) model of Ding *et al.* (1993). More aspects of the volatility models are more ambiguous. (Hansen & Lunde, 2005) Recognizing the limitations of existing ARCH specifications based on absolute returns, this study introduces the Asymmetric Power ARCH (APARCH) model. APARCH offers a more general framework that encompasses seven prior models from the literature, potentially addressing the shortcomings of these existing approaches. (Z. Ding et al., 1993) The PARCH model Taylor (1986) and Schwert (1989) advocate a standard deviation GARCH model. Compared with Bollerslev's GARCH model, this model is used to adjust the standard deviation to diminish the impact of large shocks on the conditional variance. Ding et al. (1993) further simplified the standard deviation GARCH model, naming it the power autoregressive conditional heteroscedasticity model (PARCH) (Meher, Birau, et al., 2023) The Vine copula studies effectively the complex non-linear dependence and tail dynamics among the assets. The APARCH model contains in itself volatility clustering, skewness, leptokurtosis, and long memory. This modern intervention provides more precise CoVaR estimates than older methods. The results of our analysis show that CoVaR values high sensitivity to the portfolio strategy utilized. (Mba, 2024) Often financial data shows an asymmetric volatility response, usually referred to as the leverage effect. In this case, asymmetric GARCH models that include EGARCH, TGARCH, and APARCH etc., prove instrumental. It has been vouched that the APARCH (1,2) model stands out as the optimal choice for predicting volatility. (KARANA et al., 2024) The volatility is directly associated with the rate of information flow among the financial markets which will cause fluctuation of prices in the market. The APARCH (p, q) process is stationary and entails a general class of models that includes special cases as ARCH by Engle (1982), GARCH by Bollerslev (1986), TS-GARCH by Taylor and Schwert (1986), GJR-GARCH by Glosten et al. (1993), and TARCH by Zakoian (1994). Forecasting performance of asymmetric GARCH Models (GJR and DGE), in special reference when fat-tailed asymmetric conditional distributions are

considered the conditional volatility, is better than the GARCH model.(Amin et al., 2012) Previous models were formulated to study the volatility clustering of returns but their inability to take into account the fat-tailed returns. This resulted in the need to use non-normal distributions within the GARCH models. Among them is APARCH. This model is flexibly able as it captures the fat-tails, skewness, and leverage effect characteristics of the returns.(Ogutu et al., 2018)

Now after getting a brief glimpse of the theoretical characteristics of the APARCH model let's move to applicability in the next section.

Application of APARCH Models

As Ding et al. (1993) prove and as Laurent (2004) notes, this model nests up to seven variations of the GARCH models within it subject to the δ term (e.g., a standard GARCH model has $\delta = 0$ and $\gamma = 0$).(Hartwell, 2014)They analyze Slovakia, Czech Republic, Poland, and Hungary through univariate and multivariate GARCH models that are GARCH, NGARCH, EGARCH, GJR-GARCH, AGARCH, NAGARCH and VGARCH. The findings do not reveal any asymmetric effects in the markets. Although the main finding was that strong GARCH effects are apparent for all four markets, it is revealed, that three out of seven specifications of conditional volatility are not for the market of the Czech Republic.(Ugurlu et al., 2014)There are various econometric models for forecasting and determining the daily volatility of investment funds. Here, we can highlight the specialized risk concentration forecasting models: EWMA, ARIMA, ARCH, Power GARCH, IGARCH (Integrated GARCH), APARCH, NGARCH, CGARCH, AVGARCH, SGARCH, QGARCH, MSGARCH, SETAR and others. Naturally, the GARCH models tested in the present study are not without defects.(Petrova & Todorov, 2023)Although econometric models advocate for a strong persistence of the volatility of financial returns,for a long time, short-memory models are preferred. In this paper, an extension of the ARCH(∞) model of Robinson is proposed to note the high persistence in power-transformed returns and conditional asymmetry most of the emerging markets equity indices show a stronger persistence than the standard APARCH(1,1) allow for.(Royer, 2023) After studying extensions of the APARCH model, let's narrow down ourselves to the specific theme of this article and look at Hungary, Central Europe, and the BUX Index in detail.

Hungary, Central Europe, BUX Index Volatility

High volatility can be seen in the year 2020. ATX index had high volatility in 2008 and 2020 but both the volatility was significantly similar. Results show that the return on financial assets was less volatile but high volatility in selected financial sample data series.(Kumar Meher et al., 2024b)Stock markets in Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States (G7) are more stable than in the Czech Republic, Hungary, Poland, Russia, Slovakia, and Slovenia (Central and Eastern Europe). This is because G7 markets react less to bad news compared to Central and Eastern European markets.(Égert& Koubaa, 2004)There is the existence of limited interaction between regional and global shocks, but innovations to volatility have normally a regional character.(Scheicher, 2001b)There is a time-delayed change in thevolatility of Czech and Hungarian markets due to the changes in theinterdependence of Polish, Czech, Hungarian, and Austrian markets and the capitalmarket of Germany. The delayed reaction in the Czech and Hungarian markets may be duetothe continuous adaptation of these markets to the situation in the whole group ofanalyzed markets.(Pietrzak et al., 2017)Within the Visegrad economies (Czechia, Hungary, Poland, Slovakia) hit by COVID-19 in early 2020 (March), Hungary's lockdown severity may have differed from its neighbors. Regardless, all countries experienced a decline in GDP, trade, and rising unemployment throughout 2020. Financial markets suffered similarly across the region, with a negative correlation between COVID-19 cases and the value of local currencies and stock indices. This study suggests a unified response across Visegrad economies, including Hungary, to the pandemic's financial impact.(Czech et al., 2020a)In the caseof six CEE countries (Bulgaria, Czech Republic, Hungary, Poland, Romania, and Slovenia), left-tail dependence is observed.(Mba, 2024)

It is to be noted that not very significant studies have talked specifically about the BUX index in particular in the current context but from the above significant knowledge we have gained the skeleton of the topic of this paper, now let's proceed to some advancements and gap that exists through the lens of most recent pieces of literature.

Use of machine learning algorithms like random forest using high-frequency data on unexplored sectors and areas of global stock markets to predict the stock prices or values of market indices.(Meher et al., 2024b)As the world of finance continues to become complex with time, the need for accurate and multivariate models to predict stock market volatility becomes a stark reality.(Kumar Meher et al., 2024b)Existing models for inflation persistence assume it's a constant value, failing to capture how uncertainty might affect persistence over time. This study proposes a new model, AR(1)-APARCH-ML, to address this gap. By allowing the model's inflation variance to change dynamically, it can investigate how uncertainty impacts persistence. Furthermore, the model can analyze how this "memory" of uncertainty is transmitted to inflation itself, providing a more nuanced and dynamic understanding of inflation persistence.(Canepa, 2024b)This study examines VaR for energy stocks with non-normal returns (skewed, kurtosis) and volatility clustering. It proposes a new, parsimonious L-VaR model that captures these return characteristics. To account for non-

linear return-spread dependence, various copulas (Clayton, Gumbel, etc.) are tested. The GARCH-EVT-Clayton copula emerges as the superior and most consistent model for forecasting VaR.(Kamal & Paul, 2024)Examination of the influence of climate change news on China's financial assets (stocks, bonds, exchange rates) using Climate Change News Indices (CCNIs) for various news categories (2017-2022). The research employs CCNI-return and CCNI-VaR networks to analyze connections between news and both asset returns and extreme risk. Findings suggest a moderate impact of climate change news, with specific effects on treasury and green bond returns (natural disasters, energy transition) and green bond VaR (energy transition).(Ma et al., 2024) It will be interesting to study the impact of dynamic portfolio allocation strategies on CoVaR and Δ CoVaR. This could engage exploring ML techniques to determine the most suitable portfolio allocation measures that cause tail risk.(Mba, 2024)Using deep neural networks to understand processes where data points are weakly dependent on each other (ψ -weakly dependent). They prove that the training method (empirical risk minimization) works well for this purpose and shows how quickly the network learns for various situations. Additionally, they demonstrate how this approach can be applied to classifying short sequences of data and predicting future values in specific causal models.(Kengne & Wade, 2024)It suggests that this volatility may be influenced by the mental framing of investors, as shown by their behavior in trading data. The model also accurately predicts the negative relationship between abnormal returns and ensuing volatility.(Ormos &Timotity, 2024)The results showed that a hybrid model combining EVT and ML produced better accuracy in estimating risk measures, especially in high-frequency and nonlinear data.(Melina et al., 2024) Now after seeing most recent developments in volatility estimation, let's move to specific areas of the topic concerning this article. Now following is the gap that has been identified regarding previous studies and the topic of this paper.

3. Research Gap

In the realm of financial modeling, particularly in the context of the Hungarian stock market represented by the BUX index, a significant research gap persists in the application of advanced GARCH models to accurately forecast volatility. While existing literature has explored various methodologies for volatility modeling, there remains a dearth of comprehensive studies employing advanced GARCH models tailored specifically to the dynamics of the BUX index. Despite its importance in risk management and investment decision-making, the precise volatility behavior of this index remains inadequately understood. This research endeavor seeks to bridge this gap by delving into the nuanced intricacies of BUX index volatility through the utilization of sophisticated GARCH techniques. By incorporating elements such as asymmetric effects, long memory, and time-varying parameters, this study aims to offer more precise and reliable volatility forecasts, thereby enhancing our understanding of the Hungarian stock market and facilitating more informed investment strategies.

4. Objectives of the Study

The study aims to investigate the volatility patterns of the BUX Index from 2011 to 2024. The primary objective is to formulate a robust model capable of predicting volatility by employing advanced GARCH methodologies. Specifically, the study seeks to address asymmetric volatilities stemming from significant events impacting the Hungarian market. By analyzing the intricacies of volatility dynamics within the BUX Index, the research aims to contribute to a deeper understanding of market behavior and provide valuable insights for risk management strategies and investment decision-making processes in Hungary's financial landscape.

5. Research Methodology

The nature of research is quantitative and Exploratory in nature and Econometrics tools are utilized to capture price movements. There are a total of 3216 data about the Budapest Stock Exchange (BUX) Index ranging from 07/03/2011 to 02/02/2024. The idea was to capture the volatility of the index using an appropriate Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model with the relevant distribution. The logarithmic returns were calculated using the below-mentioned formula:

$$R_t = \log \frac{P_t}{P_{t-1}}$$

Where, R_t =Logarithmic daily return for time t, P_t = ClosingPrice at time t, P_{t-1} =Corresponding price in the period at time t – 1.(Kumari et al., 2023).

First of all, the Stationarity of the data was evaluated using the Augmented Dickey-Fuller (ADF) statistic test. After the evaluation of stationarity, the ARCH Lagrange Multiplier (LM) test was applied to test the presence of heteroscedasticity in the residual series of returns. Various models from the GARCH family were employed (GARCH, TGARCH, IGARCH, EGARCH, PARCH, APARCH) across various distributions (Normal Gaussian Distribution, Student's t-distribution, Generalized Error Distribution (GED), Student's t-distribution

with fixed parameter and GED with fixed parameter) for volatility analysis. The software package used was EViews 10 for analysis.

6. Significance of the Study

The research study offers significant societal benefits by enhancing our understanding of financial market dynamics. By employing advanced GARCH models, the study provides insights into the volatility of the BUX Index, a crucial indicator of Hungary's stock market health. Understanding market volatility is vital for investors, policymakers, and financial institutions as it allows them to make informed decisions, mitigate risks, and develop effective strategies. Moreover, by unraveling the intricate patterns of volatility, the research contributes to the stability and efficiency of the financial system, fostering economic growth and prosperity. This study's findings could potentially aid in the development of more accurate forecasting models, leading to improved market regulation and investor protection. Ultimately, by shedding light on the complexities of stock market volatility, this research serves as a valuable resource for fostering a resilient and prosperous financial environment in Hungary and beyond.

7. Limitations of the Study

Although daily prices are used due to the study is a detailed account of volatility measures, due to its high statistical nature and, lack of inclusion of real-life dynamic complex variables and primary goal of finding conditional variance of returns this makes a one fit for all type framework and may not be fully competent in answering all the questions and which makes interpretation more of theoretical in nature and less comprehensible. Let's explore the limitations in an exhaustive manner:

- Limited availability of data: The data was sourced from the Hungary Stock Index database, which is freely available but due to a lack of institutional support and financial resources variable studies of intricate and micro natures were not considered. Which makes the study, a generalized piece of work.
- Generalization of results: Although APARCH accounts for the leverage factor and decay factor it is quite a flexible model encompassing seven GARCH models in itself but it is completely not comprehending all the variables of subjective nature in complex real-life scenarios.
- Model sensitivity to varied situations: APARCH indeed a complex model considering various parameters like leverage effect, Volatility clustering, decay effect etc. theoretically answers maximum queries concerning volatility but it may not be very suitable to expect answers to underlying hidden regressors specific to the scenario.
- External variables and macroeconomic regressors: External variables and market components are always unique and rarely similar but APARCH model, although a most advanced member of GARCH family but fails to account specific situational constants and variables due to high statistical and interwinded nature of specific regressors.

Now after looking at the specific nuisances of APARCH and GARCH universe let's proceed to analysis and estimation part of the paper.

8. Analysis, Estimation, and Results

To Understand price movement comprehensively we need to see the price movement visually and below is the graphical representation of actual prices:

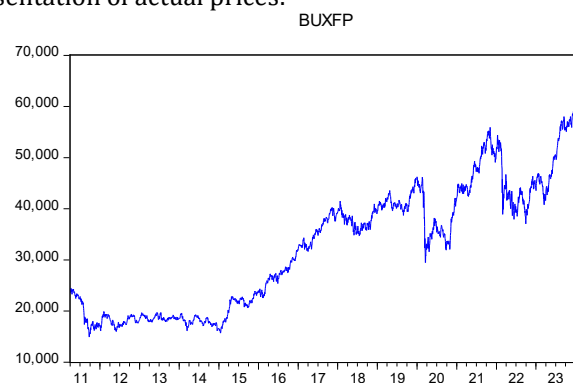


Figure 1. Daily Closing Price of Index

Source: author's Computation using EViews 10

The above is the graphical representation of the BUX Index Price over a period of almost one and half decades and there is always an increase in the price but there has a high volatile nature between 2020 and early 2023 due to various systematic and unsystematic reasons. The Budapest BUX index reached its lowest level on 19 March 2020. From 4 March to 19 March 2020, Budapest BUX decreased by approximately 47% to 29,494. The steepest daily decline in the index was approximately 12%(Czech et al., 2020b) . During the COVID-

19 pandemic, when the Visegrad currencies weakened and stock indices plunged, the volatility was higher.(Czech et al., 2020b)

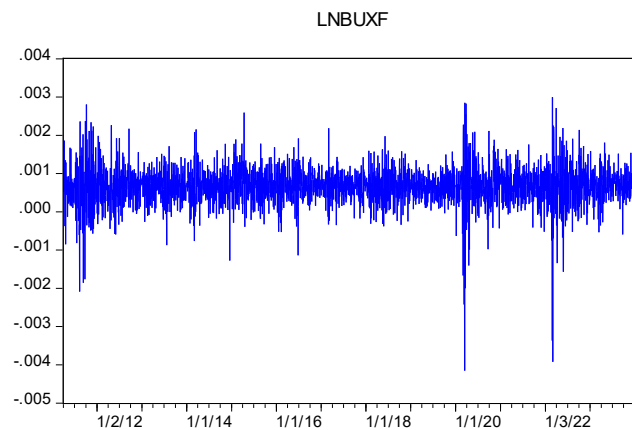


Figure 2 Log Returns Graph

Source: author's Computation using EViews 10

To make the returns stationary, Log returns are calculated. The above graphical representation of the calculated Log returns and as discussed above there exists a sharp dip between 2020 and 2022 due to various previously discussed reasons and in order to move forward with analysis test of stationarity becomes essential. Visually it appears that there are some heteroscedastic natures in the data but first test of stationarity becomes essential.

Test of Stationarity

Table 1. Augmented Dickey-Fuller Test

Null Hypothesis: LNBUXF has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic - based on SIC, maxlag=28)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-56.51756	0.0001
Test critical values: 1% level	-3.432104	
5% level	-2.862201	
10% level	-2.567165	

*MacKinnon (1996) one-sided p-values.

Source: author's computation using EViews 10

The test statistics done above is Augmented Dickey-Fuller (ADF) test and as the Probability value is less than 0.05 and test statistics is lower in value than the critical value at 5% interval, we reject the Null Hypotheses of the test and conclude that the data has not unit root and is Stationary in nature and does not possess a time-dependent structure.

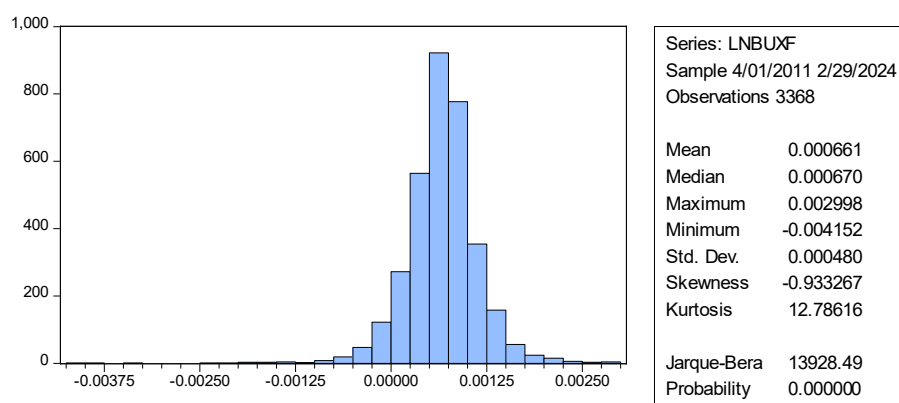


Figure 3. Test Distribution Analysis

Source: author's Computation using EViews 10

To get more acquainted with the nature of the log returns, we have the above-mentioned test statistics. We can see the data is slightly negatively skewed and along with that, the returns are leptokurtic in nature. The Kurtosis data indicates that there exists a high peak in the distribution and hence high volatility and this also indicates that most of the values are on the higher side of the mean returns value.

Heteroscedasticity Test

Table 2 ARCH effect test

Heteroskedasticity Test: ARCH

F-statistic	9.776685	Prob. F(1,3365)	0.0018
Obs*R-squared	9.754157	Prob. Chi-Square(1)	0.0018

Source: author's Computation using EViews 10

The test statistics used is ARCH LM on the residual, and the p-value we find is less than 0.05 then we conclude that there exists an ARCH effect on the residuals. This signifies there is a cluster nature in the volatility of variances.

The reason for selecting GARCH models over ARCH is because the major limitation of the ARCH Model is that tomorrow's return is an equally weighted average of the residuals squared from the last 22 days. GARCH has diminishing weights declining to zero. It provides parsimonious models that are soft to estimate and, even in its simplest form, has proven a perfect fit in forecasting conditional variance.(Meher, Puntambekar, et al., 2023)

Table 3. Decision Table

Distribution	Criteria	GARCH	IGARCH	TARCH	EGARCH	PARCH	APARCH
Normal Distribution	Akaike info criterion	-12.70358	-12.68075	-12.72389	-12.71806	-12.70300	-12.72404
	Schwarz criterion	-12.69449	-12.6753	-12.71298	-12.70715	-12.69209	-12.71131
	Log Likelihood	21391.48	21351.05	21426.67	21416.86000	21391.50000	21427.92
	ARCH significant	Yes	Yes	Yes	Yes	Yes	Yes
	Autocorrelation	No	No	No	No	No	No
	ARCH LM-Test	No	No	No	No	No	No
	GARCH significant	Yes	Yes	Yes	Yes	Yes	Yes
	significant coefficient	Yes	Yes	Yes	Yes	Yes	Yes
Student's T	Akaike info criterion	-12.74071	-12.72716	-12.75161	-12.75053	-12.74036	-12.75263
	Schwarz criterion	-12.72980	-12.71989	-12.73889	-12.73781	-12.72763	-12.73809
	Log Likelihood	21454.99	21430.18	21474.34	21472.52	21455.4	21477.05000
	ARCH significant	Yes	Yes	Yes	Yes	Yes	Yes
	Autocorrelation	No	No	No	No	No	No
	ARCH LM-Test	No	No	No	No	No	No
	GARCH significant	Yes	Yes	Yes	Yes	Yes	Yes
	significant coefficient	Yes	Yes	Yes	Yes	Yes	Yes
Generalized Error	Akaike info criterion	-12.73722	-12.72256	-12.74934	-12.74703	-12.73675	-12.74994
	Schwarz criterion	-12.72631	-12.71529	-12.73661	-12.73430	-12.72402	-12.73539
	Log Likelihood	21449.11	21422.44	21470.51	21466.63	21449.31	21472.52
	ARCH significant	Yes	Yes	Yes	Yes	Yes	Yes
	Autocorrelation	No	No	No	No	No	No
	ARCH LM-Test	No	No	No	No	No	No
	GARCH significant	Yes	Yes	Yes	Yes	Yes	Yes
	significant coefficient	Yes	Yes	Yes	Yes	Yes	Yes
T distribution (Parameter)	Akaike info criterion	-12.73936	-12.72587	-12.75150	-12.74984	-12.73890	-12.75070
	Schwarz criterion	-12.73027	-12.72041	-12.74059	-12.73893	-12.72799	-12.73797
	Log likelihood	21451.72	21427	21473.14	21470.35	21451.94	21472.81000
	ARCH significant	Yes	Yes	Yes	Yes	Yes	Yes
	Autocorrelation	No	No	No	No	No	No
	ARCH LM-Test	No	No	No	No	No	No
	GARCH significant	Yes	Yes	Yes	Yes	Yes	Yes
	significant coefficient	Yes	Yes	Yes	Yes	Yes	Yes
Generalised Error (Parameter)	Akaike info criterion	-12.73691	-12.72169	-12.74980	-12.74723	-12.73637	-12.75037
	Schwarz criterion	-12.72781	-12.71624	-12.73889	-12.73632	-12.72546	-12.73764
	Log Likelihood	21447.58	21419.97	21470.28	21470.35	21447.68	21472.24
	ARCH significant	Yes	Yes	Yes	Yes	Yes	Yes
	Autocorrelation	No	No	No	No	No	No
	ARCH LM-Test	No	No	No	No	No	No
	GARCH significant	Yes	Yes	Yes	Yes	Yes	Yes
	significant coefficient	Yes	Yes	Yes	Yes	Yes	Yes

Source: author's tabulation using MS Office

After implementing GARCH, TARCH, EGARCH, IGARCH, PARCH, and APARCH across Gaussian Normal Distribution, Student's t distribution, Generalized Error Distribution (GED), t distribution with fixed parameter and GED with fixed parameter, from the above table we can conclude that APARCH at student's t distribution is an appropriate model due to lowest Akaike info criterion (-12.75263) and comparatively lowest Schwarz criterion (-12.73809) and highest Log Likelihood (21477.05).

Table 4. APARCH (1,1) Student's t distribution

Dependent Variable: LNBUXF

Method: ML ARCH - Student's t distribution

Date: 03/06/24 Time: 18:35

Sample (adjusted): 4/06/2011 2/29/2024

Included observations: 3367 after adjustments

Convergence achieved after 74 iterations

Presample variance: backcast (parameter = 0.7)

$$@SQRT(GARCH)^C(7) = C(3) + C(4)*(ABS(RESID(-1)) - C(5)*RESID(-1))^C(7) + C(6)*@SQRT(GARCH(-1))^C(7)$$

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000640	1.36E-05	47.12812	0.0000
LNBUXF(-1)	0.039605	0.017430	2.272204	0.0231
Variance Equation				
C(3)	3.51E-07	5.55E-07	0.632463	0.5271
C(4)	0.084743	0.012161	6.968710	0.0000
C(5)	0.458331	0.098610	4.647915	0.0000
C(6)	0.887032	0.012887	68.83098	0.0000
C(7)	1.491349	0.206497	7.222151	0.0000
T-DIST. DOF	7.934804	1.026219	7.732080	0.0000
R-squared	0.000338	Mean dependent var		0.000661
Adjusted R-squared	0.000041	S.D. dependent var		0.000480
S.E. of regression	0.000480	Akaike info criterion		-12.75263
Sum squared resid	0.000777	Schwarz criterion		-12.73809
Log likelihood	21477.05	Hannan-Quinn criter.		-12.74743
Durbin-Watson stat	2.023871			

Source: author's Computation using Eviews 10

Ding, Granger, and Engle (1993) brought the APARCH model (asymmetric power ARCH), which increased two parameters based on the GARCH model. One of the parameters is used to measure the Leverage Effect.(D. Ding, n.d.)The APARCH model can well explain fat tails, excess kurtosis, and leverage effects. The model is as follows:

$$y_t = x_t \xi + \varepsilon_t$$

$$t = 1, 2 \dots T$$

$$\sigma_t^\delta = \omega + \sum_{j=1}^q \alpha_j (|\varepsilon_{t-j}| - \gamma_j \varepsilon_{t-j})^\delta + \sum_{i=1}^p \beta_i (\sigma_{t-i})^\delta$$

$$\varepsilon_t = \sigma_t z_t, z_t \sim N(0, 1)$$

$$k(\varepsilon_{t-j}) = |\varepsilon_{t-j}| - \gamma_j \varepsilon_{t-j}$$

The APARCH equation has to satisfy the following conditions:

1. $\omega > 0, \alpha_j \geq 0, j = 1, 2, \dots, q, \beta_i \geq 0, i = 1, 2, \dots, p$ when $\alpha_j = 0, j = 1, 2, \dots, q, \beta_i = 0, i = 1, 2, \dots, p$, then $\sigma_t^2 = \omega$. Due to this the variance is positive, so $\omega > 0$.
2. $0 \leq \sum_{i=1}^q \beta_i \leq 1$

Table 5. APARCH Coefficients and their meanings

Notation	Meaning
ω C(3)	A constant term representing the baseline of volatility
α C(4)	Persistence of Volatility/Positive Shock
γ C(5)	Leverage Effect
β C(6)	Persistence of Volatility/Negative Shock
δ C(7)	Decay Rate/Power Term

Source: authors' tabulation using MS Office

From the above APARCH analysis, it could be figured out that ω is not statistically significant, this represents that there is no strong baseline effect and the movement of the returns is dependent on dynamics of stock such as shocks, leverage effect, asymmetric effects, and volatility. We see the value of α representing

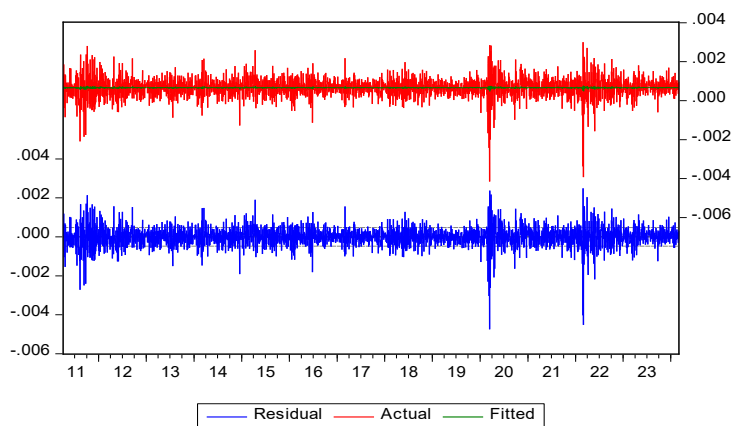
persistence of positive shocks at 0.084743 value signifies the influence of positive shocks but not at a very significant level, it may be due to not very significant boosting factor in the past one and half decades.

Similarly, if we look at the β value representing the persistence of negative shocks and its effects on volatility with a value of 0.887032, which is very much closer to 1 represents the high influence of negative shocks on overall volatility. This may be due to the COVID-19 pandemic, Political Unrest in the recent past poor macroeconomic parameters such as low GDP growth, inflation, etc, and current major geopolitical events and mechanisms.

After looking at the persistence of positive and negative shocks on overall volatility let's look at the δ value representing the Power term/decay value of 0.000 000 0351, which is very much closer to 0, meaning a very slow decay rate signifying that the effect of shocks are long-lasting and slow fading resulting in a high amount of volatility. That signifies that the shocks experienced during pandemic and minor shocks due to external and macro level events are still one of the causes of volatility.

As discussed before, negative shocks have a higher influence than positive shocks which signifies a leverage effect is confirmed by 0.458331 γ value. This signifies asymmetric results to the news and volatility clustering resulting in long terms of persistent high volatility. Now after getting the mathematical understanding of volatility let's look at graphical representation of asymmetric fitted graph and forecast of variance graph for greater understanding.

Figure 4. Graphical representation of estimated volatility



Source: author's Computation using EViews 10

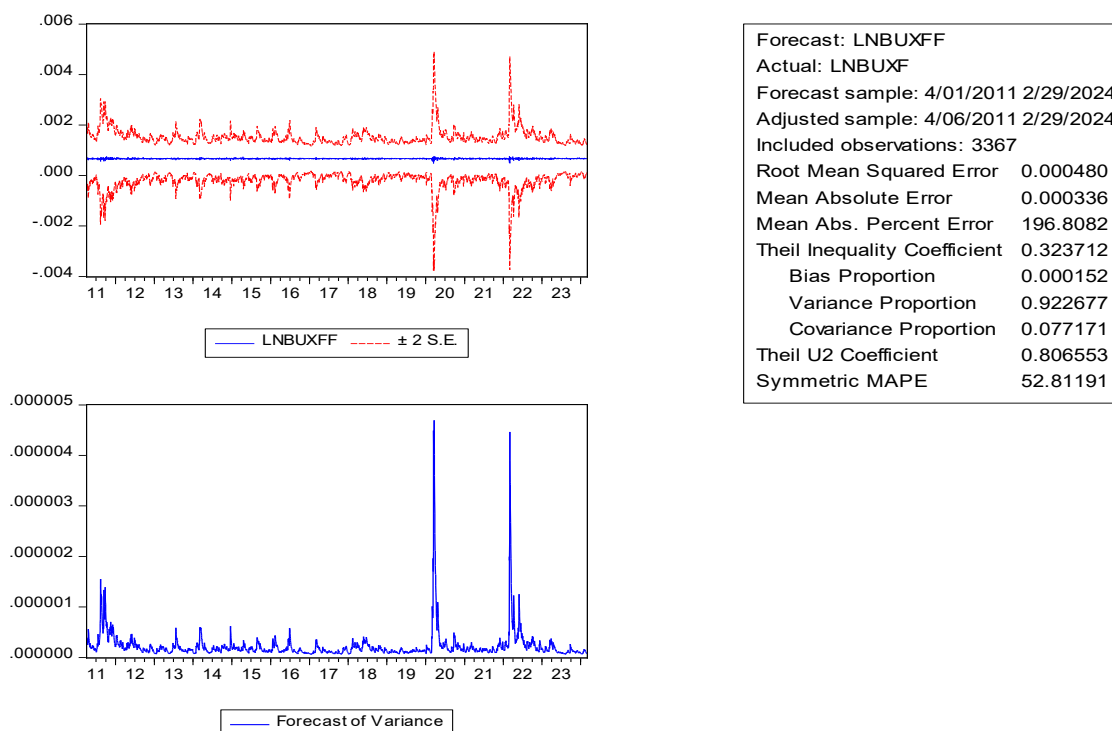


Figure 5 Graphical representation of Forecast of Prices Returns and Volatility

Source: author's Computation using EViews 10

From the graphical representation we are easily able to see that the residuals graph very well resembles the actual volatility meaning the model has been successful in capturing the volatility. The graphs very well explain the nature of volatility i.e. small volatility with along with some high volatility. Variance is also almost constant with small fluctuations along with few high fluctuations. There is also a strong chance in the future that the nature of the volatility will be similar.

5. Conclusion and Recommendations

APARCH (1,1) seems to be a fit model to study the volatility of the BUX Index as through the variance equation, it has been made clear that there exists an asymmetry, volatility clustering, slower decay, and high influence of positive shock and a low response to positive shocks. Along with these, it has also been reported that there is no baseline effect. Similar results can also be concluded with real-life events that have governed the BUX Index, including the Greek financial crisis, Intervention by the European Union, macroeconomic disturbances, Boost in the healthcare and technology sector, the COVID-19 pandemic, and geopolitical disturbances as clear from small volatility flowed by a huge volatility in 2020 and 2022.

GARCH, TGARCH, IGARCH, EGARCH, PARCH, and APARCH were tested along six distributions and APARCH with Student's t distribution was deemed fit according to the results based upon AIC, SC, and Log Likelihood criterion. This signifies that there is the presence of an asymmetry, leverage effect, volatility clustering, and time-based decay factor (Power).

The APARCH (1,1) model has been successful in explaining the volatility of the BUX Index with daily data of almost 15 years and it is able to answer most of the queries that may arise while researching the movement of the BUX Index but there still exists a scope of more variable and factor-based analysis. However, with an analysis with this model, it could be concluded that APARCH is a flexible model at a certain level and incorporates in itself the nature of all basic GARCH models at specific conditions and also resembles FIGARCH and GJR-GARCH in a certain sense.

The nature of volatility that has been discovered from the analysis creates a ripple effect of information and awareness among both investors and policymakers as the index is reflective of the economy of Hungary as a whole and the BUX Index is one the most preferred investment avenue for investors as far as Hungarian investment is concerned. The study also leaves the scope for future analysis and research about individual shocks (positive/negative), delayed decay, and leverage effect individually in detail to provide a more informed and micro-level analysis on BUX. The nature of the volatility of BUX is unique in its own sense and can be a case to study index-based research and future forecasting with more advanced models. Although, the autocorrelation models measuring conditional variance like APARCH (1,1) is a complex model in itself but given the complex nature of multiple variables networking and producing results, there remains a scope of more detailed agent and stakeholder-based study through copulas, VaR, Machine Learning etc.

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